Noise self-pumping in long Josephson junctions

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The noise self-pumping effect in a spatially extended system modeled by a sine-Gordon equation is investigated. This effect is realized in a similar fashion as usual ac self-pumping effect in long Josephson junctions: fluctuating solitons, radiating from the junction, induce fluctuating magnetic field, which in turn modulates the dynamics of the soliton chain, and increases the spectral linewidth. Contrary to the theory for short Josephson junctions, predicting linear decrease in the spectral linewidth with increase in junction length, the minimum of the linewidth versus the length is observed both for uniform and nonuniform bias feed distributions.

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I. INTRODUCTION

For several decades the sine-Gordon equation, describing a viscous flow of solitary waves under the action of bias force, has been the most adequate model for the long Josephson tunnel junctions (JTJs). This model has been used for the description of Josephson oscillators,[1–3] vortex transistors,[4] Josephson transmission lines,[5] and also in DNA-promoter dynamics,[6,7] dislocation theory, and charge density waves in dielectrics (see Ref. 8 and references therein). The basic properties of long underdamped JTJ are well known.[5] However, the investigation of fluctuational properties of this spatially extended object is restricted by its complexity. Even the mean escape times from zero-voltage state have been investigated analytically and very approximately; the task can hardly be solved numerically, since it requires numerical solution of nonlinear partial differential equation with noise.[9] For example, there were discussions about the kink nucleation rate in annular JTJ (see Ref. 8). For JTJ of linear geometry with only one fluxon traveling and rejecting from boundaries (shuttle fluxon oscillator), the formula for the spectral linewidth gives reasonable agreement with the experimental results.[1] For multifluxon regimes the dynamics is much more complex. One of the examples of the device, where the multifluxon regime is maintained by applying large external magnetic field, is the flux-flow oscillator (FFO).[2] It demonstrates very rich behavior, which in particular results in the ac self-pumping effect,[3,10] where the radiation of the FFO leads to specific steps at current-voltage (IV) curves. The FFO has emerged as the most promising local oscillator for superconducting spectrometers.[11] It was a puzzling problem for more than a decade to find a formula for the spectral linewidth of FFO: it was intensively studied both experimentally[10–14] and theoretically.[15–19] It is known that the FFO linewidth is rather large, and by roughly one order of magnitude exceeds the linewidth of a short JTJ.[20] The formula for the linewidth,[18] taking into account the differential resistance as a function of both bias current and magnetic field, has proven to adequately describe the experimental results.[13,14] Still, the conversion of bias current fluctuations to magnetic field fluctuations and the reason of so large linewidth is unclear. In Ref. 21, the investigation of the influence of the bias current profile of FFO on its power and the linewidth has been performed. It has been found that for different current profiles, giving significantly different differential resistances, the minimal attainable linewidth is nearly the same so one cannot go below the certain limit. On the other hand, varying the load resistances of the FFO, the linewidth can be decreased further by about 1.5 times. On the basis of these two facts, the hypothesis of the noise self-pumping effect has been discussed.

The aim of the present paper is to study the noise self-pumping effect in a long JTJ described by the one-dimensional (1D) sine-Gordon equation with noise, in particular, by investigation of the spectral characteristics versus junction length, and to discover the reason of anomalously large FFO linewidth.

II. STATEMENT OF THE PROBLEM

The sine-Gordon equation, giving a good qualitative description of basic properties of a long JTJ, has the form

$$\phi_{tt} + \alpha\phi_t - \phi_{xx} = \beta\phi_{xxt} + \eta(x) - \sin(\phi) + \eta_f(x,t), \qquad (1)$$

where indices t and x denote temporal and spatial derivatives. Space and time are normalized to the Josephson penetration length λ_J and to the inverse plasma frequency ω_p^{-1}, respectively, $\alpha=\omega_p/\omega_c$ is the damping parameter, $\omega_p=\sqrt{2eI_c/\hbar C}$, $\omega_c=2eI_cR_N/\hbar$, I_c is the critical current, C is the JTJ capacitance, R_N is the normal state resistance, β is the surface loss parameter, $\eta(x)$ is the dc overlap bias current density, normalized to the critical current density J_c, and $\eta_f(x,t)$ is the fluctuational current density. If the critical current density is fixed and the fluctuations are treated as white Gaussian noise with zero mean, its correlation function is $\langle \eta_f(x,t)\eta_f(x',t')\rangle = 2\alpha\gamma\delta(x-x')\delta(t-t')$, where $\gamma=I_T/(J_c\lambda_J)$ is the dimensionless noise intensity (Ref. 9), $I_T=2ekT/\hbar$ is the thermal current, e is the electron charge, $\hbar$ is the Planck constant, k is the Boltzmann constant, and T is the temperature.

The boundary conditions that describe coupling to the environment have the form[3]

$$\phi(0,t)_x + r_Lc_L\phi(0,t)_{xt} - c_L\phi(0,t)_{tt} + \beta r_Lc_L\phi(0,t)_{xtt} + \beta\phi(0,t)_{xt} = \Gamma - \Delta\Gamma, \qquad (2)$$

$$\phi(L,t)_x + r_R c_R \phi(L,t)_{xt} + c_R \phi(L,t)_{tt} + \beta r_R c_R \phi(L,t)_{xtt} + \beta\phi(L,t)_{xt} = \Gamma + \Delta\Gamma. \quad (3)$$

Here Γ is the normalized magnetic field, $\Delta\Gamma=0.05\Gamma$ (see Ref. 3) and L is the dimensionless length of JTJ. The terms with the dimensionless capacitances and resistances $c_{L,R}$ and $r_{L,R}$ are the RC load of a JTJ placed at the left (output) and at the right (input) ends, respectively. Following Ref. 19, if both the overlap $\eta_{ov}=(1/L)\int_0^L \eta(x)dx$ and the inline $\eta_{in}=2\Delta\Gamma/L$ components of the current are present, the total current η_t, with respect to which all current-voltage characteristics (IVCs) $v(\eta_t)$ will be computed is the sum of overlap and inline components: $\eta_t=\eta_{ov}+\eta_{in}$.

For the linewidth of short JTJ, the following formula is known[20] (here and below the linewidth is defined as full width, half power):

$$\Delta f_s = 2\alpha\gamma r_d^2/L, \quad (4)$$

where $r_d=dv/d\eta_t$ is the differential resistance. It is found experimentally[10] that even for small r_d, the FFO linewidth is almost one order of magnitude larger than predicted by Eq. (4). The formula for the FFO linewidth of Ref. 18, which in addition to r_d takes into account the differential resistance over magnetic field $r_d^{CL}=Ldv/d\Gamma$,

$$\Delta f_{\mathrm{FFO}} = 2\alpha\gamma(r_d + \sigma r_d^{CL})^2/L, \quad (5)$$

demonstrates good agreement with the experiment.[13,14] In this formula however, the factor σ, describing the conversion of bias current fluctuations to magnetic field fluctuations, is unknown, and the nature of this conversion is not yet clear.[13,14,18,19] To study the FFO linewidth, let us perform computer simulation of Eq. (1) with noise. The used implicit difference scheme was tested in Ref. 9.

In the frame of the present paper, let us consider and compare two limiting cases of bias current distribution: uniform $\eta(x)=\eta_0$ and nonuniform one, characteristic for a superconductive thin film:[20,22]

$$\eta(x) = \frac{\eta_0 L}{\pi\sqrt{x(L-x)}}. \quad (6)$$

The current profile [Eq. (6)] naturally appears in a long narrow JTJ, and only special means, such as the increase in the junction width, allow to make the distribution more uniform.

III. RESULTS

It is reasonable to take the parameters for practical FFOs with long flux flow steps (FFSs):[3] $\alpha=0.033$, $\beta=0.035$, $c_L=c_R=100$, $r_L=2$, $r_R=100$, and $\gamma=0.1$. The IVCs as well as the linewidth are computed for 1000 realizations for sequentially increasing the bias current values with fixed magnetic field $\Gamma=3.6$. From Fig. 1, one can see that for uniform bias current distribution, the Fiske steps with an increase in the junction length merge into continuous FFS. After approaching $L\approx 20$ the IVCs almost coincide with each other. A different picture is observed for nonuniform distribution [Eq. (6)]; while for small lengths, the behavior is nearly the same as for uniform; for larger lengths the height of the FFSs decreases (see the curve for $L=40$), and differential resistance r_d grows significantly.

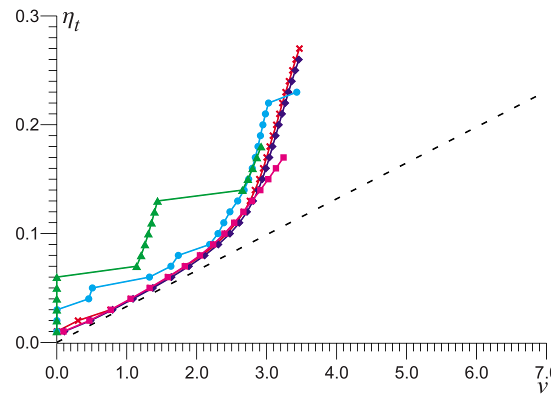

FIG. 1. (Color online) The IVC of loaded FFO for $\alpha=0.033$, $\beta=0.035$, $\Gamma=3.6$, and $\gamma=0.1$. Magenta curve with rectangles—bias current distribution [Eq. (6)] for $L=40$, all other curves are for uniform distribution. Green curve with triangles: $L=2$, cyan curve with circles: $L=5$, red curve with crosses: $L=20$, and blue curve with diamonds: $L=40$. By dashes, the ohmic line is shown.

The power spectral density of FFO is computed as Fourier transform of the correlation function of the second kind $\Phi[\tau]=\frac{1}{T_{av}}\int_0^{T_{av}}\langle v_0(t)v_0(t+\tau)\rangle dt$, where $v_0(t)=d\varphi(t,0)/dt$ is the voltage at the RC load ($x=0$), and T_{av} is the averaging time. There are two general restrictions, complicating the calculation of the spectral density: on one hand, the time step should be small enough to resolve oscillations, and on the other hand, the averaging time T_{av} should be rather large to resolve fine spectral spikes. Due to these restrictions, the noise intensity was chosen—$\gamma=0.1$. Nevertheless, this is the same limit of low noise intensity as in the experiments, since IVCs are almost unaffected by the noise, the spectral spikes are narrow, and the linewidth perfectly scales proportionally to the noise intensity (see below).

In Fig. 2 the power spectral density of FFO is presented. As one can see, the emitted signal of FFO at flux-flow steps

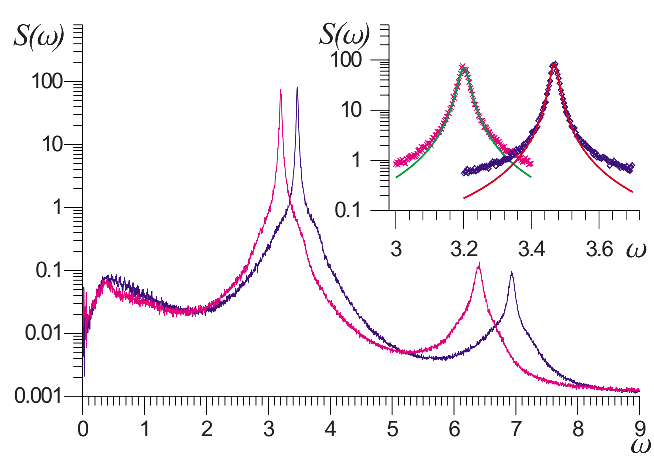

FIG. 2. (Color online) The calculated spectral density for $L=40$, $\Gamma=3.6$, and $\gamma=0.1$: for uniform bias current distribution $\eta_t=0.24$—blue (right) curve; for distribution [Eq. (6)], $\eta_t=0.16$—magenta (left) curve. Inset: the same, enlarged around the spikes; Lorentzian approximation—red and green solid curves.

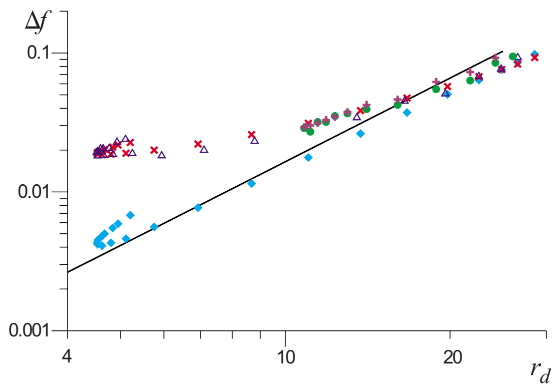

FIG. 3. (Color online) FFO linewidth versus differential resistance for $L=40$ and $\gamma=0.1$. Empty triangles and crosses—simulations and theory (5), $\sigma=0.185$ for uniform bias current distribution; circles and daggers—simulations and theory (5), $\sigma=0.11$ for nonuniform distribution [Eq. (6)]; cyan diamonds—theory of Ref. 17 for uniform distribution; solid line—theory (4).

(FFSs) is nearly sinusoidal, in agreement with Ref. 23 and experimental results: the power contained in the second and third harmonics is much lower than in the main one. Also, the spectral peak is perfectly Lorentzian in about two orders of magnitude interval. This is quite different with spectral densities at the displaced linear slope, i.e., at small bias currents and magnetic fields, where chaotic behavior is possible and the linewidth is extremely large.[12] It is important to note that, while r_d is about two times larger and the FFS is lower in the nonuniform case, the spectral density, computed at the top of the step, has nearly the same power, and only 1.5 times larger linewidth than in the uniform case. This is important for practical applications, where power consumption can be reduced significantly with the cost of moderate linewidth increase.

In Fig. 3 the FFO linewidth versus differential resistance r_d for junctions of $L=40$ and $\gamma=0.1$ is presented for uniform and nonuniform [Eq. (6)] bias current distributions. It is seen that in both cases, formula (5) is in good agreement with numerical results, while formula (4) for short JTJ and formula from Ref. 17 [in formula (4), α must be substituted by inverse static resistance η_t/v] significantly underestimate the linewidth. Certain increase in the linewidth for small r_d range is due to the slight growth of r_d at the top of IV curves. The minimum of Δf corresponds nearly to the top of IVCs, which is often used as working point for practical applications. In spite of more than two times different values of r_d for uniform and nonuniform [Eq. (6)] bias current distributions, the minimal attainable linewidth is nearly the same, which makes the FFO to be quite different from other types of Josephson oscillators, such as the short JTJ [Eq. (4)] and shuttle fluxon oscillator.[1] Taking into account that the considered model is 1D and the only bias current is fluctuating, this is clear indication of the noise self-pumping effect in the FFO, which is realized in a similar fashion as usual ac self-pumping effect:[3,10] fluctuating fluxons, radiating from the FFO, induce fluctuating magnetic field at the FFO ends, which in turn modulates the fluxon dynamics, and increases the linewidth [boundary conditions (2) and (3) depend on the phase, which fluctuates since it is governed by Eq. (1) with noise]. This important contribution into the magnetic field fluctuations has been missed in Ref. 18 due to the method used: first, the dynamical equations without noise were derived, after which small fluctuational variations of parameters were calculated. If the same approach in Ref. 18 is considered with noise from the very beginning, this additional "self-pumping" contribution to the magnetic field fluctuations naturally appears.

By the change of variables, the small parameter $\epsilon=(\alpha/\eta)^2 \ll 1$ in front of the $\sin(\phi)$ term can be introduced. Then the solution of Eq. (1) can formally be obtained as the expansion: $\phi(x,t)=\phi_0(x,t)+\epsilon\phi_1(x,t)+\epsilon^2\phi_2(x,t)+\ldots$, even in the case when the bias current and correspondingly the phase ϕ are fluctuating. The noise must be taken into account in the equation for $\phi_0(x,t)$,

$$\phi_{0_{tt}}+\alpha\phi_{0_t}-\phi_{0_{xx}}-\beta\phi_{0_{xxt}}=\eta(x)+\eta_f(x,t), \tag{7}$$

since even vanishing noise leads to the diffusion of the phase. This diffusion, consequently leads to nonzero linewidth[24] of the signal $\phi_1(x,t)$, and the right-hand side (RHS) of Eq. (8), which are narrow-band random processes, centered around oscillation frequency ω_J [here $\phi_{2s}(x,t)$ is the low frequency part of $\phi_2(x,t)$],

$$\phi_{1_{tt}}+\alpha\phi_{1_t}-\phi_{1_{xx}}-\beta\phi_{1_{xxt}}=-\sin(\phi_0+\phi_{2s}+\ldots). \tag{8}$$

It is obvious that the product of two narrow-band processes in the RHS of the equation for $\phi_2(x,t)$,

$$\phi_{2_{tt}}+\alpha\phi_{2_t}-\phi_{2_{xx}}-\beta\phi_{2_{xxt}}=-\phi_{1p}\cos(\phi_0+\phi_{2s}+\ldots), \tag{9}$$

gives the correction to the low frequency part of the spectrum and the narrow-band process at $2\omega_J$.

Due to linearity of Eq. (7), supplemented by the boundary conditions (2) and (3), it is seen that the phase can be cast into the sum: $\phi(x,t)=\phi_d(x,t)+\phi_f(x,t)$, where indices d and f stand for the deterministic and fluctuational parts, respectively. However, it is still a challenge to solve Eqs. (7)–(9) with so complex boundary conditions even in the small noise limit. Therefore, we can only assume that the noise in the boundary conditions [which is the sum of all terms with $\phi_f(x,t)$] is proportional to the bias current fluctuations with a certain unknown factor σ, and applying further the same approach in Ref. 18, one can obviously get the formula (5). In doing this, we of course neglect by filtering of noise contributions in the boundary conditions, but in the limit of small noise the system does not feel the noise color if its cut-off frequency is much larger than the linewidth.[24] As discussed in Ref. 18, an additional parametric broadening of the linewidth[24] could be due to the multiplication of bias current and magnetic field fluctuations on the second and higher harmonics of FFO radiation, but due to their small power (see Fig. 2), this contribution can be neglected as well.

Let us investigate how the above observed effect varies with the junction length L. In the theory of short JTJ the noise intensity is inversely proportional to the critical current. For long JTJ the critical current increases with the increase in the length so one may think that longer junctions with constant critical current density will have smaller linewidth [see Eq. (4)]. The corresponding curves are presented

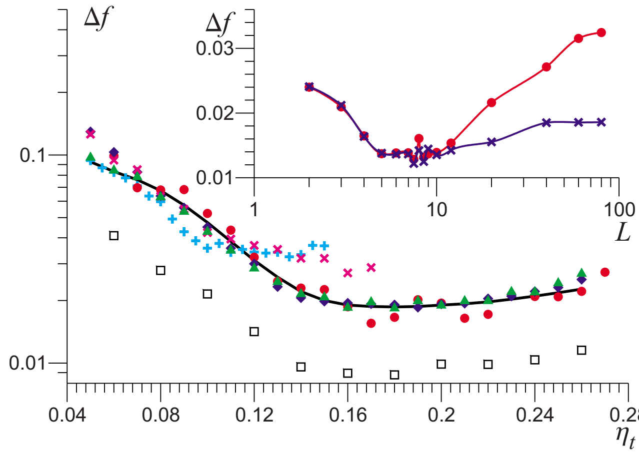

FIG. 4. (Color online) The FFO linewidth versus total current for α=0.033, β=0.035, and Γ=3.6. Uniform distribution γ=0.1: circles—L=20; diamonds—L=40; triangles—L=80; black solid curve—theory (5) for L=60 and σ=0.17. Empty black rectangles—uniform distribution for L=40 and γ=0.05. Nonuniform distribution γ=0.1: magenta crosses—L=40; cyan daggers—L=80. Inset: the minimal attainable linewidth as function of length γ=0.1: blue crosses—uniform distribution, and red circles—nonuniform distribution.

in Fig. 4. It is seen that in the uniform case, the increase in L for $L\geq 40$ does not help to decrease the linewidth, the curves nearly coincide; while for the nonuniform case, the linewidth increases for large lengths. This means that the discrepancy with formula (4) will grow for $L\to\infty$. The lowest curve is computed for noise intensity γ=0.05, and Δf is two times smaller than for γ=0.1, confirming that we are indeed in a low noise limit, and the corresponding curves for smaller noise intensity can be obtained by scaling. In the inset of Fig. 4, the minimal attainable linewidth is presented versus junction length L for uniform and nonuniform bias current distributions. Both curves have minima at $L\approx 5-10$. For $L\geq 40$, the minimal linewidth in the uniform case is 1.5–2 times smaller than in the nonuniform one.

To further demonstrate that the noise self-pumping is mainly spatial effect, let us consider an unloaded ($c_L=c_R$ =0) long JTJ with $\beta=\Delta\Gamma=0$ for uniform bias current distribution. Then the boundary conditions take the simplest form: $\phi(0,t)_x=\phi(L,t)_x=\Gamma$. The corresponding IVCs are given in Fig. 5 for α=0.2, Γ=3, γ=0.05, and L=1,5,20. The cyan curve is shown for β=0.035, L=20 for comparison. From Fig. 6, one can see that while for L=1 the computer simulation results nearly agree with the formula for short JTJ [Eq. (4)], with increase in the length the difference increases, and for L=20, the discrepancy is about four times. Nearly the same difference between simulations and theory (4) is observed for β=0.035, L=20. Therefore, we can conclude that the noise self-pumping is indeed spatial effect and not induced by coupling to the environment or surface losses.

Finally, let us consider the plots of the linewidth versus junction length for several values of bias current η_t =0.64;0.7;0.74 for the unloaded junction with α=0.2, β =0, Γ=3, and γ=0.05 (see Fig. 7). The curves as functions of L also have minima but in comparison with the minimal attainable linewidth (inset of Fig. 4), the minima are deeper,

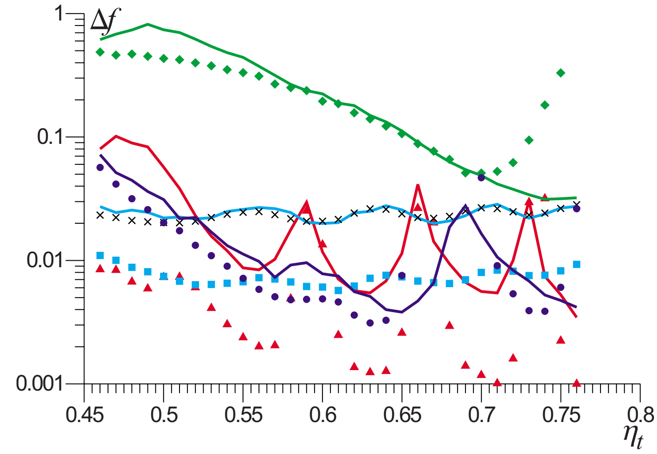

FIG. 6. (Color online) The linewidth of unloaded FFO versus total current for uniform bias current distribution α=0.2, Γ=3, and γ=0.05. Solid curves—simulations, symbols—theory (4). For β =0: green diamonds—L=1; blue circles—L=5; red triangles—L =20. Cyan rectangles—β=0.035, and L=20. Black crosses—theory (5) for β=0.035, L=20, and σ=0.2.

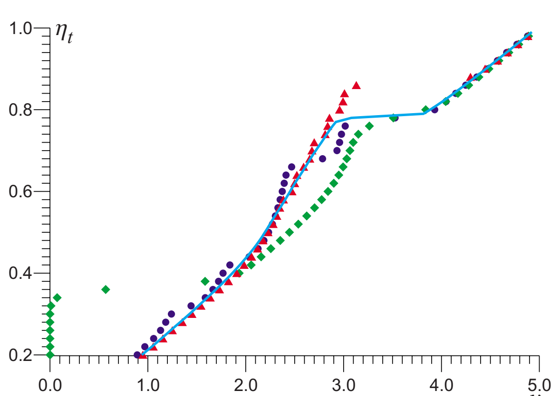

FIG. 5. (Color online) The IVC of unloaded FFO for α=0.2, Γ=3, and γ=0.05. For β=0: green diamonds: L=1; blue circles: L=5; red triangles: L=20. Cyan curve: β=0.035, and L=20.

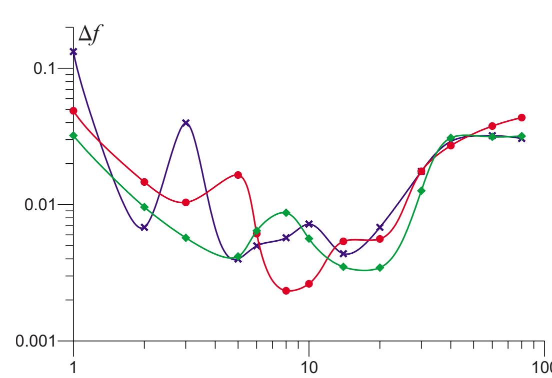

FIG. 7. (Color online) The linewidth of unloaded FFO vs length for different values of bias current (α=0.2,β=0,Γ=3,γ=0.05): η_t=0.64—blue crosses, η_t=0.7—red circles, and η_t=0.74—green diamonds.

which is explained by the presence of more steep Fiske steps (see Fig. 5) for intermediate lengths of the junction.

IV. CONCLUSIONS

The spectral properties of a long Josephson tunnel junction described by the one-dimensional sine-Gordon equation with noise have been studied. The noise self-pumping effect in this spatially extended system is investigated. This effect is realized in a similar fashion as usual ac self-pumping effect in long JTJ: fluctuating solitons, radiating from the junction, induce fluctuating magnetic field, which in turn modulates the dynamics of the soliton chain, and increases the spectral linewidth. Contrary to the theory for short JTJ, predicting linear decrease in the spectral linewidth with increase in junction length, the minimum of the linewidth versus the length is observed both for uniform and nonuniform bias feed distributions. Nearly perfect agreement of the computer simulation results with the formula (5) has been achieved.

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